

SECOND EDITION

**SIGNALS
&
SYSTEMS**

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1.6 BASIC SYSTEM PROPERTIES

In this section we introduce and discuss a number of basic properties of continuous-time and discrete-time systems. These properties have important physical interpretations and relatively simple mathematical descriptions using the signals and systems language that we have begun to develop.

1.6.1 Systems with and without Memory

A system is said to be *memoryless* if its output for each value of the independent variable at a given time is dependent only on the input at that same time. For example, the system specified by the relationship

$$y[n] = (2x[n] - x^2[n])^2 \quad (1.90)$$

is memoryless, as the value of $y[n]$ at any particular time n_0 depends only on the value of $x[n]$ at that time. Similarly, a resistor is a memoryless system; with the input $x(t)$ taken as the current and with the voltage taken as the output $y(t)$, the input-output relationship of a resistor is

$$y(t) = Rx(t), \quad (1.91)$$

where R is the resistance. One particularly simple memoryless system is the *identity system*, whose output is identical to its input. That is, the input-output relationship for the continuous-time identity system is

$$y(t) = x(t),$$

and the corresponding relationship in discrete time is

$$y[n] = x[n].$$

An example of a discrete-time system with memory is an *accumulator* or *summer*

$$y[n] = \sum_{k=-\infty}^n x[k], \quad (1.92)$$

and a second example is a *delay*

$$y[n] = x[n-1]. \quad (1.93)$$

A capacitor is an example of a continuous-time system with memory, since if the input is taken to be the current and the output is the voltage, then

$$y(t) = \frac{1}{C} \int_{-\infty}^t x(\tau) d\tau, \quad (1.94)$$

where C is the capacitance.

Roughly speaking, the concept of memory in a system corresponds to the presence of a mechanism in the system that retains or stores information about input values at times

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other than the current time. For example, the delay in eq. (1.93) must retain or store the preceding value of the input. Similarly, the accumulator in eq. (1.92) must "remember" or store information about past inputs. In particular, the accumulator computes the running sum of all inputs up to the current time, and thus, at each instant of time, the accumulator must add the current input value to the preceding value of the running sum. In other words, the relationship between the input and output of an accumulator can be described as

$$y[n] = \sum_{k=-\infty}^{n-1} x[k] + x[n], \quad (1.95)$$

or equivalently,

$$y[n] = y[n-1] + x[n]. \quad (1.96)$$

Represented in the latter way, to obtain the output at the current time n , the accumulator must remember the running sum of previous input values, which is exactly the preceding value of the accumulator output.

In many physical systems, memory is directly associated with the storage of energy. For example, the capacitor in eq. (1.94) stores energy by accumulating electrical charge, represented as the integral of the current. Thus, the simple RC circuit in Example 1.8 and Figure 1.1 has memory physically stored in the capacitor. Similarly, the automobile in Figure 1.2 has memory stored in its kinetic energy. In discrete-time systems implemented with computers or digital microprocessors, memory is typically directly associated with storage registers that retain values between clock pulses.

While the concept of memory in a system would typically suggest storing *past* input and output values, our formal definition also leads to our referring to a system as having memory if the current output is dependent on *future* values of the input and output. While systems having this dependence on future values might at first seem unnatural, they in fact form an important class of systems, as we discuss further in Section 1.6.3.

1.6.2 Invertibility and Inverse Systems

A system is said to be *invertible* if distinct inputs lead to distinct outputs. As illustrated in Figure 1.45(a) for the discrete-time case, if a system is invertible, then an *inverse system* exists that, when cascaded with the original system, yields an output $w[n]$ equal to the input $x[n]$ to the first system. Thus, the series interconnection in Figure 1.45(a) has an overall input-output relationship which is the same as that for the identity system.

An example of an invertible continuous-time system is

$$y(t) = 2x(t), \quad (1.97)$$

for which the inverse system is

$$w(t) = \frac{1}{2}y(t). \quad (1.98)$$

This example is illustrated in Figure 1.45(b). Another example of an invertible system is the accumulator of eq. (1.92). For this system, the difference between two successive

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